PB 154685

TECHNICAL REPORT
WVT-RR-6015-R

ELASTIC-PLASTIC ANALYSIS OF CLOSED-END TUBES

BY

C. D. SUTHERLAND

R. E. WEIGLE

JULY 1960

D. A. PROJECT NO. 501-01-033

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LIST OF SYMBOLS

r, θ, z	=	Coordinate axes
σ_z^e, σ_z^p	=	Longitudinal stress in elastic and plastic regions
σ_r^e, σ_r^p	=	Radial stress in elastic and plastic regions
$\sigma_\theta^e, \sigma_\theta^p$	=	Circumferential stress in elastic and plastic regions
k	=	Yield stress in simple shear
$\bar{\sigma}_z^e, \bar{\sigma}_z^p$	=	$\sigma_z^e/2k$, $\sigma_z^p/2k$
$\bar{\sigma}_r^e, \bar{\sigma}_r^p$	=	$\sigma_r^e/2k$, $\sigma_r^p/2k$
$\bar{\sigma}_\theta^e, \bar{\sigma}_\theta^p$	=	$\sigma_\theta^e/2k$, $\sigma_\theta^p/2k$
μ	=	Modulus of rigidity
a	=	Internal (bore) radius
b	=	External radius
$\bar{b}$	=	b/a
$\tilde{b}$	=	$\ln(\sqrt{3}\,\bar{b}/2)$
ρ	=	Radius to elastic-plastic interface
$\bar{\rho}$	=	ρ/a
$\tilde{\rho}$	=	$\ln(\sqrt{3}\,\bar{\rho}/2)$
$\bar{r}$	=	r/a
$\tilde{r}$	=	$\ln(\sqrt{3}\,\bar{r}/2)$
p	=	Internal pressure
p^*	=	Pressure for incipient plastic flow
p^{**}	=	Pressure for complete plasticity
$\bar{p}, \bar{p}^*, \bar{p}^{**}$	=	$p/2k$, $p^*/2k$, $p^{**}/2k$

ELASTIC-PLASTIC ANALYSIS OF CLOSED-END TUBES

ABSTRACT

An approximate solution is derived for the stresses in a circular, closed-end, unrestrained tube subjected to an internal pressure of sufficient magnitude to induce plastic flow. The Mises yield criterion is assumed in the plastic region of the tube. The radial and circumferential stresses are found to be in good agreement with those for the plane stress case. Results are shown graphically.

Cross-reference Data

Artillery	01
Basic Research	99
Studies & Investigations	01-01
	99-01
Cannon	
Stress Analysis	

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CONCLUSIONS

By assuming a logarithmic distribution for the longitudinal stress in the plastic portion of the tube wall and comparing these results with other numerically obtained solutions for the pressurized, closed-end tube, it has been found that very little difference in the radial and tangential stress distribution is observed. Of course, the end-force must be equilibrated by the longitudinal stresses acting across the tube thickness, but as long as this condition is met it appears that any assumed distribution for the σ_z stress in the plastic region, has little effect upon the aforementioned radial and tangential stresses.

One point which should be mentioned is that the Mises yield criterion was considered to govern the behavior of the tube material in the plastic state. Hence, the fact that the σ_r and σ_θ stresses do not differ greatly, regardless of what end conditions for the tube are considered, is not totally unexpected. Particular reference is made to the solution of this same problem wherein the Tresca yield criterion is utilized. In this case, it is found that the end condition has no influence upon σ_θ and σ_r .

Comparison of these same stresses with the plane strain solution[1] indicates approximate agreement. This is not surprising in view of the fact that it can be readily shown that the magnitude of the end restraint, which is required to maintain the plane strain condition, approaches the end force produced in the closed-end tube for the case of complete plasticity[2].

A fair agreement with the plane stress results is also obtained[3]. In fact, for the plane strain, plane stress and closed-end conditions, it is interesting to note that very little difference is observed in the distribution and magnitude of the radial and tangential stresses for the case of initial yielding at the bore surface.

In comparing the magnitude of the longitudinal stress σ_z , as developed herein, with the values of σ_z obtained by use of the Prandtl-Reuss stress strain relations for a perfectly plastic material, large discrepancies are observed. Furthermore, it is generally agreed that use of the Prandtl-Reuss relation leads to more accurate stress predictions; however, the mathematical difficulties encountered are more formidable.

CONCLUSIONS (CONT.)

In any event, useful stress predictions for σ_θ and σ_r can be obtained by the methods which are outlined in this report.

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INTRODUCTION

The problem is one of a thick-walled, circular, closed end, unrestrained tube subjected to an internal pressure of sufficient magnitude to induce plastic flow. The plastic deformation occurs first at the inner surface of the tube and, as the pressure is increased, the plastic region continues to grow until the entire tube becomes plastic. A state of contained plasticity is assumed. The tube material is considered to be homogeneous, isotropic, compressible, and perfectly plastic.

To describe the pressure induced stresses and strains in the tube, a system of cylindrical coordinates (r, θ, z) is employed, where the z-axis coincides with the axis of the tube. All stresses and strains are assumed to be independent of θ and z.

FORMULATION

The principal directions of stress at a generic point of the tube are radial, circumferential and axial, and the only equation of equilibrium which remains to be satisfied is[4]:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{1}$$

We denote the inner surface of the tube by $r = a$, the outer surface by $r = b$, and the interface between the elastic and plastic region by $r = \rho$. We denote by σ_r^e, σ_θ^e, σ_z^e and σ_r^p, σ_θ^p, σ_z^p the stresses in the elastic and plastic regions, respectively. The Mises yield criterion is assumed and hence, throughout the plastic region $a \leqslant r \leqslant \rho$, $a \leqslant \rho \leqslant b$, the following condition must be satisfied:

$$(\sigma_r^p - \sigma_\theta^p)^2 + (\sigma_\theta^p - \sigma_z^p)^2 + (\sigma_z^p - \sigma_r^p)^2 = 6k^2 \tag{2}$$

where k is the yield stress in simple shear for the material.

The solution for the elastic region $\rho \leq r \leq b$, $a \leq \rho \leq b$, is given by the Lamé equations:

$$\sigma_r^e = A + 2\mu D - 2\mu B/r^2$$

$$\sigma_\theta^e = A + 2\mu D + 2\mu B/r^2$$

$$\sigma_z^e = A \qquad (3)$$

The boundary conditions to be satisfied are:

1. $\sigma_r = -p \quad , \quad r = a$

2. $\sigma_r = 0 \quad , \quad r = b$

3. $\sigma_r^e = \sigma_r^p \ , \ \sigma_\theta^e = \sigma_\theta^p \ , \ \sigma_z^e = \sigma_z^p \ , \ r = \rho$

4. $p\pi a^2 = 2\pi \int_a^b \sigma_z \, r\,dr = 2\pi \left\{ \int_a^\rho \sigma_z^p \, r\,dr + \int_\rho^b \sigma_z^e \, r\,dr \right\}$

The last condition states that the axial stress integrated over the thickness of the tube equilibrates the force acting on the closed end of the tube. These conditions are sufficient to determine the constants of integration and establish a relationship between p and ρ.

To determine the value of $p = p^*$ for incipient plastic flow, i.e. $\rho = a$, we apply the boundary conditions 1, 2, and 4 to equations (3) and find the constants A, B, D in terms of p. This yields:

$$A = \frac{p^* a^2}{b^2 - a^2} \ , \quad B = \frac{p^* a^2 b^2}{2\mu(b^2 - a^2)} \ , \quad D = 0 \, .$$

We next apply the Mises yield condition, equation (2), setting $r = a$, and solve for p^* :

$$p^* = \frac{k(b^2 - a^2)}{b^2}$$

To determine the plastic stress state for $\rho > a$, we first solve (2) for σ_θ^p :

$$\sigma_\theta^p = \frac{1}{2}\left\{\sigma_r^p + \sigma_z^p \pm \sqrt{3}\sqrt{4k^2 - (\sigma_r^p - \sigma_z^p)^2}\right\} \qquad (4)$$

The ambiguous sign $(\pm)$ in (4) is determined by using the results for incipient plastic flow, evaluating the stresses for $r = a$, and substituting for the stresses in (4). It follows that the plus sign should be used in (4).

We introduce the notations:

$$\bar{\sigma}_r = \frac{\sigma_r}{2k}, \quad \bar{\sigma}_\theta = \frac{\sigma_\theta}{2k}, \quad \bar{\sigma}_z = \frac{\sigma_z}{2k}, \quad \bar{A} = \frac{A}{2k}$$

$$\bar{B} = \frac{B}{2k}, \quad \bar{D} = \frac{D}{2k}, \quad \bar{P} = \frac{P}{2k}$$

The equation of equilibrium and equation (4) become:

$$\frac{d\bar{\sigma}_r}{dr} + \frac{\bar{\sigma}_r - \bar{\sigma}_\theta}{2r} = 0 \qquad (1)$$

$$\bar{\sigma}_\theta^p = \frac{1}{2}\left\{\bar{\sigma}_r^p + \bar{\sigma}_z^p + \sqrt{3\left[1 - (\bar{\sigma}_r^p - \bar{\sigma}_z^p)^2\right]}\right\} \qquad (4)$$

Substituting for $\bar{\sigma}_\theta^p$ from (4) into (1), and letting

$y = \bar{\sigma}_r^{\,p} - \bar{\sigma}_z^{\,p}$ we obtain

$$\frac{dy}{dr} + \frac{1}{2r}\left\{ y - \sqrt{3(1-y^2)} \right\} + \frac{d\bar{\sigma}_z^{\,p}}{dr} = 0 \tag{5}$$

Letting

$$\frac{dy}{dr} - \frac{\sqrt{3(1-y^2)}}{2r} = 0 \tag{6}$$

and

$$\frac{d\bar{\sigma}_z^{\,p}}{dr} + \frac{y}{2r} = 0 \tag{7}$$

two equations are obtained which are immediately integrable. Integrating (6), we find:

$$y = \bar{\sigma}_r^{\,p} - \bar{\sigma}_z^{\,p} = \sin\left[\frac{\sqrt{3}}{2} \ln\left(\frac{r}{c_1}\right) \right] \tag{8}$$

where C_1 is a constant of integration. Substituting for y from (8) into (7) and integrating, we obtain:

$$\bar{\sigma}_z^{\,p} = \frac{\sqrt{3}}{3} \cos\left[\frac{\sqrt{3}}{2} \ln\left(\frac{r}{c_1}\right) \right] + C_2 \tag{9}$$

where C_2 is a constant of integration. From (8) and (4), we now find:

$$\bar{\sigma}_r^{\,p} = \sin\left[\frac{\sqrt{3}}{2} \ln\left(\frac{r}{c_1}\right) \right] + \frac{\sqrt{3}}{3} \cos\left[\frac{\sqrt{3}}{2} \ln\left(\frac{r}{c_1}\right) \right] + C_2 \tag{10}$$

and

$$\overline{\sigma}_\theta{}^P = \frac{1}{2}\sin\left[\frac{\sqrt{3}}{2}\ln\left(\frac{r}{c_1}\right)\right] + \frac{5\sqrt{3}}{6}\cos\left[\frac{\sqrt{3}}{2}\ln\left(\frac{r}{c_1}\right)\right] + c_2 \quad (11)$$

Applying boundary conditions 1, 2, 3, 4, we obtain:

$$\sin\left[\frac{\sqrt{3}}{2}\ln\left(\frac{\rho}{c_1}\right)\right] - 2\mu\overline{D} + \frac{2\mu\overline{B}}{\rho^2} = 0 \quad (12)$$

$$\frac{1}{2}\sin\left[\frac{\sqrt{3}}{2}\ln\left(\frac{\rho}{c_1}\right)\right] + \frac{5\sqrt{3}}{6}\cos\left[\frac{\sqrt{3}}{2}\ln\left(\frac{\rho}{c_1}\right)\right] + c_2 - \overline{A} - 2\mu\overline{D} - \frac{2\mu\overline{B}}{\rho^2} = 0 \quad (13)$$

$$\frac{\sqrt{3}}{3}\cos\left[\frac{\sqrt{3}}{2}\ln\left(\frac{\rho}{c_1}\right)\right] + c_2 - \overline{A} = 0 \quad (14)$$

$$\overline{A} + 2\mu\overline{D} - \frac{2\mu\overline{B}}{b^2} = 0 \quad (15)$$

$$\overline{p} + \sin\left[\frac{\sqrt{3}}{2}\ln\left(\frac{a}{c_1}\right)\right] + \frac{\sqrt{3}}{3}\cos\left[\frac{\sqrt{3}}{2}\ln\left(\frac{a}{c_1}\right)\right] + c_2 = 0 \quad (16)$$

$$-\frac{a^2\overline{p}}{2} + \frac{c_2}{2}(\rho^2 - a^2) + \frac{4\sqrt{3}}{57}\left\{\rho^2\left[\frac{\sqrt{3}}{2}\sin\left(\frac{\sqrt{3}}{2}\ln\frac{\rho}{c_1}\right) + 2\cos\left(\frac{\sqrt{3}}{2}\ln\frac{\rho}{c_1}\right)\right]\right\}$$

$$-\frac{4\sqrt{3}}{57}\left\{a^2\left[\frac{\sqrt{3}}{2}\sin\left(\frac{\sqrt{3}}{2}\ln\frac{a}{c_1}\right) + 2\cos\left(\frac{\sqrt{3}}{2}\ln\frac{a}{c_1}\right)\right]\right\} + \frac{\overline{A}}{2}(b^2 - \rho^2) = 0 \quad (17)$$

These six equations involve the six unknown quantities $\overline{A}$, $\overline{B}$, $\overline{D}$, c_1, c_2 and $\overline{p}$. The first four of these equations are used to eliminate $\overline{A}$, $\overline{B}$, $\overline{D}$ and c_2 from the last two. After simplification and the introduction of the notations

$$\overline{\rho} = \rho/a, \quad \overline{b} = b/a, \quad \tilde{\rho} = \frac{\sqrt{3}}{2}\ln\overline{\rho}, \quad \tilde{b} = \frac{\sqrt{3}}{2}\ln\overline{b}, \quad \tilde{c}_1 = -\frac{\sqrt{3}}{2}\ln\left(\frac{c_1}{a}\right)$$

these two equations take the form:

$$\sin\tilde{c}_1\left\{-\sqrt{3}\left[3(19-15\bar{b}^2)\tilde{\rho}^2+19\bar{b}^2(3\bar{b}^2-7)\right]\sin\tilde{\rho}+48\bar{b}^2+3\left[(3\bar{b}^2-19)\tilde{\rho}^2+57\bar{b}^2(\bar{b}^2-1)\right]\cos\tilde{\rho}\right\}+228\bar{b}^2\bar{p}$$

$$+\cos\tilde{c}_1\left\{3\left[(3\bar{b}^2-19)\tilde{\rho}^2+57\bar{b}^2(\bar{b}^2-1)\right]\sin\tilde{\rho}+64\sqrt{3}\,\bar{b}^2+\sqrt{3}\left[3(19-15\bar{b}^2)\tilde{\rho}^2+19\bar{b}^2(3\bar{b}^2-7)\right]\cos\tilde{\rho}\right\}=0 \qquad (18)$$

$$\sin\tilde{c}_1\left\{-\sqrt{3}(3\tilde{\rho}^2-7\bar{b}^2)\sin\tilde{\rho}-3(3\bar{b}^2+\tilde{\rho}^2)\cos\tilde{\rho}+12\bar{b}^2\right\}+12\bar{b}^2\bar{p}$$

$$+\cos\tilde{c}_1\left\{-3(3\bar{b}^2+\tilde{\rho}^2)\sin\tilde{\rho}+\sqrt{3}(3\tilde{\rho}^2-7\bar{b}^2)\cos\tilde{\rho}+4\sqrt{3}\,\bar{b}^2\right\}=0 \qquad (19)$$

Equations (18) and (19) are then solved for $\sin\tilde{c}_1$ and $\cos\tilde{c}_1$ in terms of p and ρ ($\bar{p}$ and $\tilde{\rho}$), and the relationship between p and ρ is then determined from the identity $\cos^2\tilde{c}_1+\sin^2\tilde{c}_1=1$.

The resulting expression is:

$$\bar{p}=\left[76\bar{b}^2\Delta\right]\left\{2\sqrt{57}\,\bar{b}^2\left[-8\sqrt{3}(\tilde{\rho}^2-\bar{b}^2)\sin\tilde{\rho}-32\bar{b}^2\cos\tilde{\rho}+3\tilde{\rho}^4-6\tilde{\rho}^2\bar{b}^2+19\bar{b}^4+16\right]^{\frac{1}{2}}\right\}^{-1} \qquad (20)$$

where

$$76\bar{b}^2\Delta=\left\{-4\left[3(\bar{b}^2+1)\tilde{\rho}^2+\bar{b}^2(13-19\bar{b}^2)\right]\sin\tilde{\rho}-16\sqrt{3}\left[(\bar{b}^2-1)\tilde{\rho}^2+2\bar{b}^2\right]\cos\tilde{\rho}+\sqrt{3}\left[3\tilde{\rho}^4-6\tilde{\rho}^2\bar{b}^2+19\bar{b}^4+16\bar{b}^2\right]\right\} \qquad (21)$$

If we let $\rho=a$, then $\bar{\rho}=1$, $\tilde{\rho}=\frac{\sqrt{3}}{2}\ln\bar{\rho}=0$ and (20) reduces to the previously determined result for incipient plastic flow:

$$\bar{p}^*=\frac{\bar{b}^2-1}{2\bar{b}^2}$$

For $\rho = b$, the entire tube is in the plastic state and the pressure corresponding to this, $\bar{p} = \bar{p}^{**}$, is found from (20) by letting $\bar{\rho} = \bar{b}$, $\tilde{\rho} = \tilde{b}$.

$$\bar{p}^{**} = \frac{2\sqrt{57}}{57} \frac{\left\{\sqrt{3}\,(\bar{b}^2+1)(1-\cos\tilde{b}) + 4(\bar{b}^2-1)\sin\tilde{b}\right\}}{\left\{\bar{b}^4 + 1 - 2\bar{b}^2\cos\tilde{b}\right\}^{1/2}}$$

The values of the various constants are given by:

$$\sin\tilde{c}_1 = \bar{b}^2\left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{-3(\bar{\rho}^2+19\bar{b}^2)\sin\tilde{\rho} + \sqrt{3}(15\bar{\rho}^2-19\bar{b}^2)\cos\tilde{\rho} + 4\sqrt{3}\right\}$$

$$\cos\tilde{c}_1 = \bar{b}^2\left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{\sqrt{3}(15\bar{\rho}^2-19\bar{b}^2)\sin\tilde{\rho} + 3(\bar{\rho}^2+19\bar{b}^2)\cos\tilde{\rho} - 60\right\}$$

$$C_2 = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{4(3\bar{\rho}^2+13\bar{b}^2)\sin\tilde{\rho} - 16\sqrt{3}(\bar{\rho}^2-2\bar{b}^2)\cos\tilde{\rho} - \sqrt{3}(3\bar{\rho}^4-6\bar{\rho}^2\bar{b}^2+19\bar{b}^4)\right\}$$

$$\bar{A} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{12(\bar{\rho}^2+4\bar{b}^2)\sin\tilde{\rho} - 4\sqrt{3}(4\bar{\rho}^2-3\bar{b}^2)\cos\tilde{\rho} - \sqrt{3}\,\bar{\rho}^2(3\bar{\rho}^2-7\bar{b}^2)\right\}$$

$$\bar{B} = \frac{\bar{b}^2}{2\mu}\left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{12\bar{\rho}^2\sin\tilde{\rho} - 16\sqrt{3}\,\bar{\rho}^2\cos\tilde{\rho} - \sqrt{3}\,\bar{\rho}^2(3\bar{\rho}^2-19\bar{b}^2)\right\}$$

$$\bar{D} = \frac{6\bar{b}^2}{\mu}\left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\left\{-4\sin\tilde{\rho} - \sqrt{3}\cos\tilde{\rho} + \sqrt{3}\,\bar{\rho}^2\right\}$$

Introducing the notation $\bar{r} = r/a$, $\tilde{r} = \frac{\sqrt{3}}{2}\ln\bar{r}$ we find the stresses for the plastic region $a \le r \le \rho$, $a \le \rho \le b$:

$$\overline{\sigma}_r^{\,p} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{-4\bar{b}^2(3\bar{\rho}^2-19\bar{b}^2)\sin(\tilde{r}-\tilde{\rho})+16\sqrt{3}\,\bar{\rho}^2\bar{b}^2\cos(\tilde{r}-\tilde{\rho})$$

$$-16\bar{b}^2(4\sin\tilde{r}+\sqrt{3}\cos\tilde{r})+4(3\bar{\rho}^2+13\bar{b}^2)\sin\tilde{\rho}$$

$$-16\sqrt{3}(\bar{\rho}^2-2\bar{b}^2)\cos\tilde{\rho}-\sqrt{3}(3\bar{\rho}^4-6\bar{\rho}^2\bar{b}^2+19\bar{b}^4)\Big\}$$

$$\overline{\sigma}_\theta^{\,p} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{-4\bar{b}^2(9\bar{\rho}^2-19\bar{b}^2)\sin(\tilde{r}-\tilde{\rho})+2\sqrt{3}\,\bar{b}^2(5\bar{\rho}^2+19\bar{b}^2)\cos(\tilde{r}-\tilde{\rho})$$

$$-8\bar{b}^2(5\sin\tilde{r}+6\sqrt{3}\cos\tilde{r})+4(3\bar{\rho}^2+13\bar{b}^2)\sin\tilde{\rho}$$

$$-16\sqrt{3}(\bar{\rho}^2-2\bar{b}^2)\cos\tilde{\rho}-\sqrt{3}(3\bar{\rho}^4-6\bar{\rho}^2\bar{b}^2+19\bar{b}^4)\Big\}$$

$$\overline{\sigma}_z^{\,p} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{-\bar{b}^2(15\bar{\rho}^2-19\bar{b}^2)\sin(\tilde{r}-\tilde{\rho})+\sqrt{3}\,\bar{b}^2(\bar{\rho}^2+19\bar{b}^2)\cos(\tilde{r}-\tilde{\rho})$$

$$-4\bar{b}^2(\sin\tilde{r}+5\sqrt{3}\cos\tilde{r})+4(3\bar{\rho}^2+13\bar{b}^2)\sin\tilde{\rho}$$

$$-16\sqrt{3}(\bar{\rho}^2-2\bar{b}^2)\cos\tilde{\rho}-\sqrt{3}(3\bar{\rho}^4-6\bar{\rho}^2\bar{b}^2+19\bar{b}^4)\Big\}$$

For the elastic region $\rho \leqslant r \leqslant b$, $a \leqslant \rho \leqslant b$, the stresses are given by:

$$\overline{\sigma}_r^{\,e} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{12\sin\tilde{\rho}-16\sqrt{3}\cos\tilde{\rho}-\sqrt{3}(3\bar{\rho}^2-19\bar{b}^2)\Big\}\bar{\rho}^2\frac{(\bar{r}^2-\bar{b}^2)}{\bar{r}^2}$$

$$\overline{\sigma}_\theta^{\,e} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{12\sin\tilde{\rho}-16\sqrt{3}\cos\tilde{\rho}-\sqrt{3}(3\bar{\rho}^2-19\bar{b}^2)\Big\}\bar{\rho}^2\frac{(\bar{r}^2+\bar{b}^2)}{\bar{r}^2}$$

$$\overline{\sigma}_z^{\,e} = \left[\frac{\bar{p}}{76\,\bar{b}^2\Delta}\right]\Big\{12(\bar{\rho}^2+4\bar{b}^2)\sin\tilde{\rho}-4\sqrt{3}(4\bar{\rho}^2-3\bar{b}^2)\cos\tilde{\rho}$$

$$-\sqrt{3}\,\bar{\rho}^2(3\bar{\rho}^2-7\bar{b}^2)\Big\}$$

RESULTS

Numerical results were obtained for the case $\bar{b} = b/a = 2$, $\bar{\rho} = \rho/a = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$
The equations for $\bar{p} = p/2k$ and the various stresses were coded for the IBM 650 digital calculator. The results are shown graphically in figures one through five.

ACKNOWLEDGEMENT

The authors wish to express their appreciation for the assistance given by Mr. Paul J. Loatman, Chief, Computer Unit, in obtaining the numerical results required for this report.

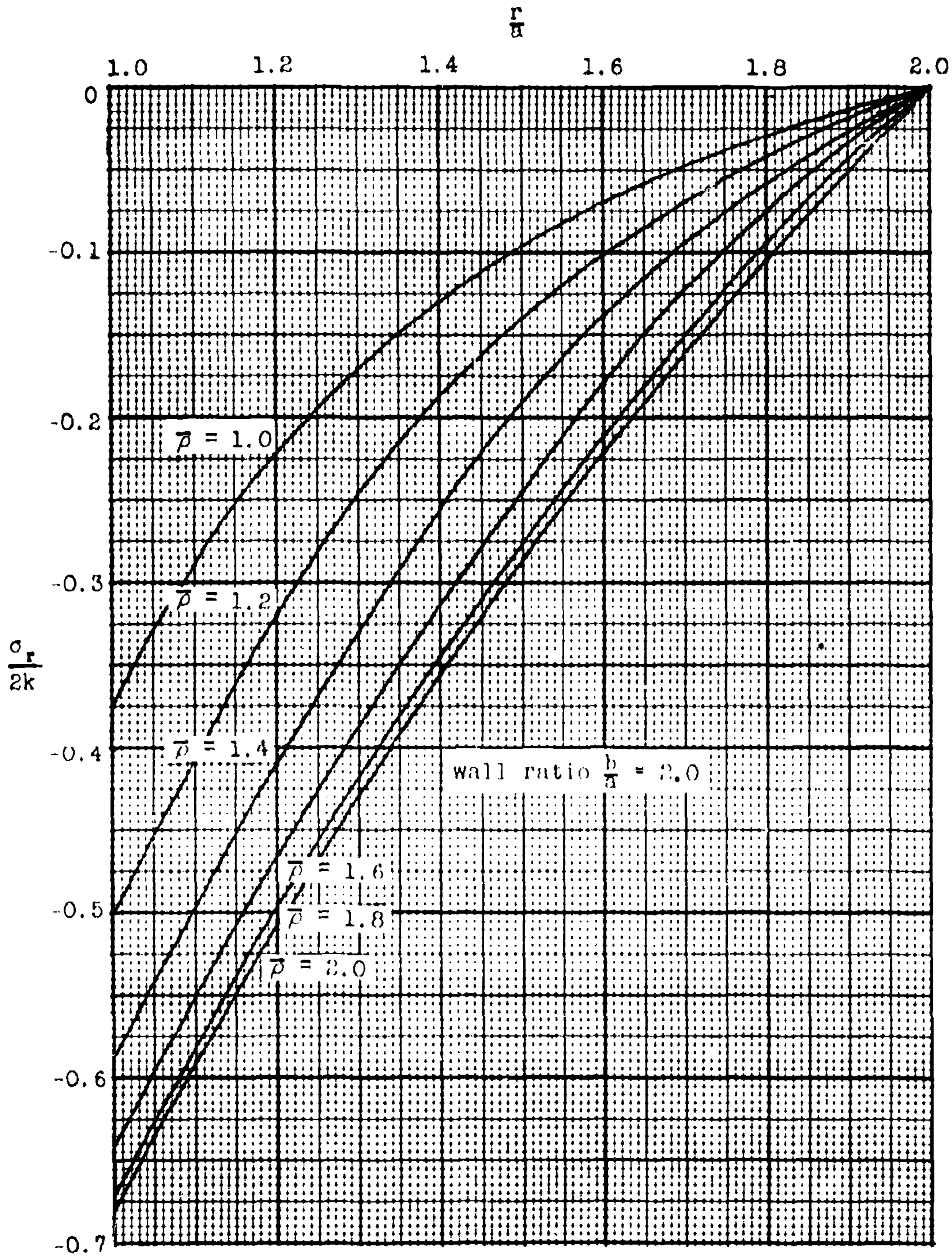

Figure 1. $\frac{\sigma_r}{2k}$ vs. $\frac{r}{a}$.

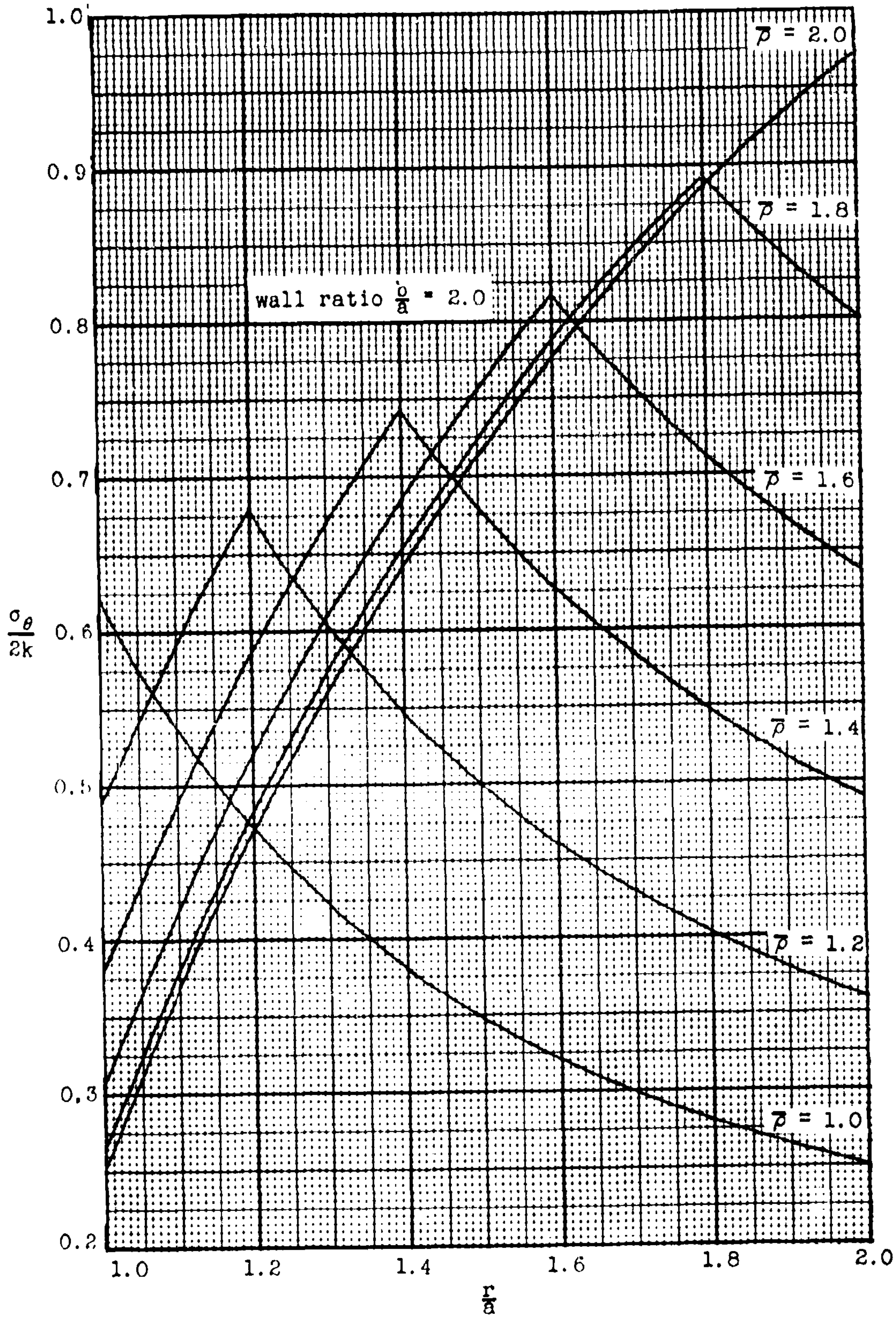

Figure 2. $\frac{\sigma_\theta}{2k}$ vs. $\frac{r}{a}$.

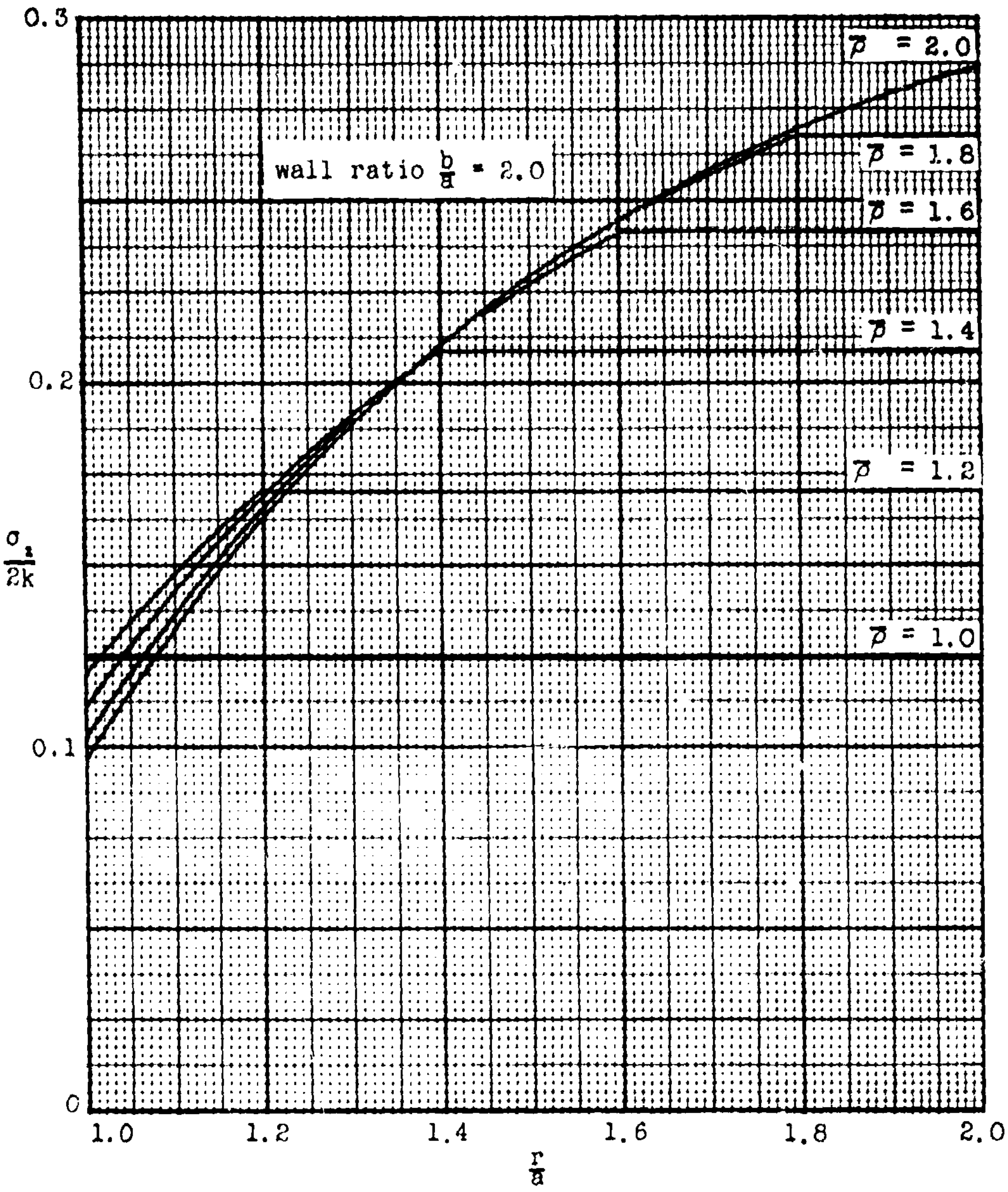

Figure 3. $\frac{\sigma_z}{2k}$ vs. $\frac{r}{a}$.

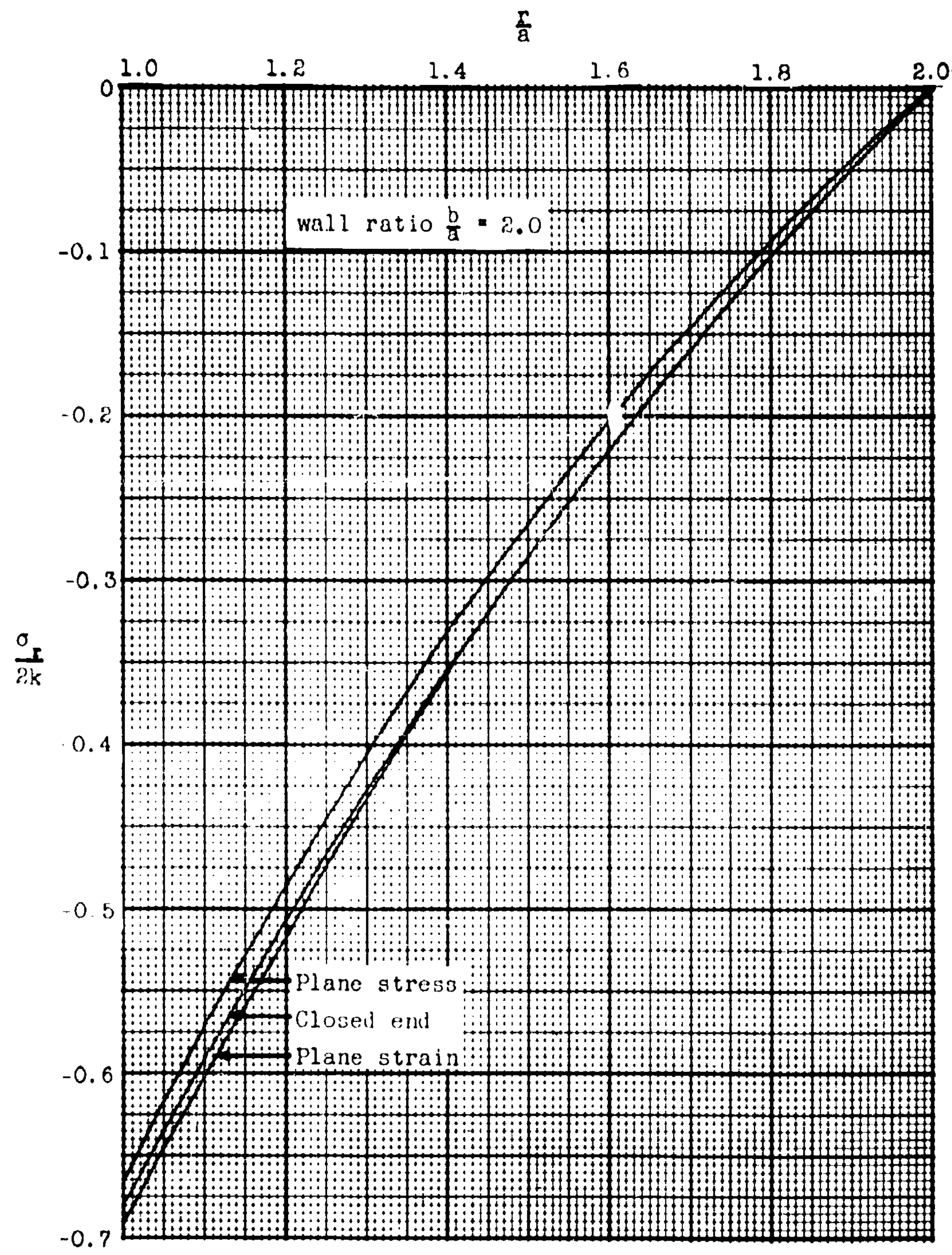

Figure 4. $\frac{\sigma_r}{2k}$ vs. $\frac{r}{a}$ for $\rho = b$ for the plane strain, plane stress and closed end cases.

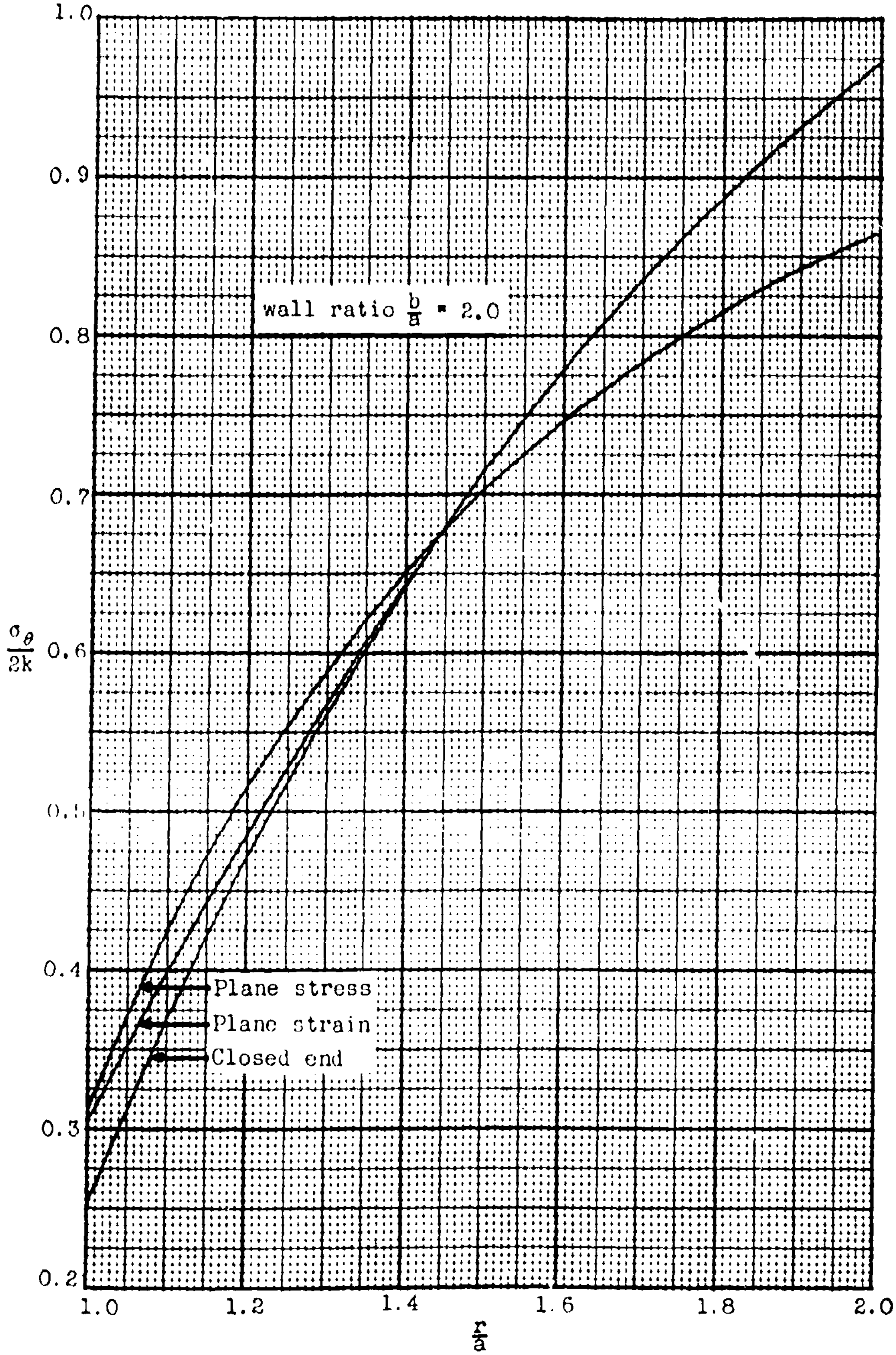

Figure 5. $\frac{\sigma_\theta}{2k}$ vs. $\frac{r}{a}$ for $\rho = b$ for the plane strain, plane stress and closed end conditions.

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